ECE 388 – Automatic Control

Nyquist Plot

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Compulsory Course in Electronic and Communication Engineering
Credits (2/2/3)

Course Webpage: http://ECE388.cankaya.edu.tr

Nyquist Plot: Transfer Function

Pole-Zero Representation

$$G(s) = K \frac{(s - z_1)\cdots(s - z_m)}{s^q(s - p_{q+1})\cdots(s - p_n)}$$

- $G(s)$ has $0 \leq q \leq n$ poles at zero and $n - q$ poles different from zero

Example

Gap 1

Nyquist Plot: Transfer Function

Assumption

- $G(s)$ is proper: $n \geq m$

Observation

- $G(s)$ is a complex number for all $s \in \mathbb{C}$

Example

Gap 2

⇒ **Plot $G(s)$ in the complex plane along a well-defined path for s**

Nyquist Plot: Construction

Closed Path $\mathcal{C}$ in Complex Plane

- $\mathcal{C}$ includes the imaginary axis
- $\mathcal{C}$ encircles all poles of G_o on the imaginary axis by a small semi-circle in the right-half plane
- $\mathcal{C}$ closes by a large semi-circle in the right-half plane
- $\mathcal{C}$ is traversed in clockwise direction
  ⇒ **Such path $\mathcal{C}$ encircles all poles of $G(s)$ in the right-half plane**

Illustration: s-Plane

Gap 3

Nyquist Plot: Path Representation

Components of the Closed Path $\mathcal{C}$

- Imaginary axis: $s = j\omega$ for $-\infty \leq \omega \leq \infty$
- Large semi-circle: $s = R \cdot e^{j\Phi}$ for $\pi/2 \geq \Phi \geq -\pi/2$ and R large
- Small semi-circles for each p_i on imaginary axis: $s = p_i + r \cdot e^{j\varphi}$ for $-\pi/2 \leq \varphi \leq \pi/2$ and r small

Nyquist Curve

- Defined by mapping $s \mapsto G_o(s)$ for all $s \in \mathcal{C}$
  $\Rightarrow$ Each $s \in \mathcal{C}$ is mapped to the complex number $G_o(s)$

Nyquist Plot (NP)

- Graphical representation of the Nyquist curve in the complex plane
  $\rightarrow$ We call the complex plane G_o-plane
  $\rightarrow$ Complex number $G_o(s)$ is plotted in the G_o-plane for each $s \in \mathcal{C}$

Nyquist Plot: Simple Example

$$G(s) = \frac{1}{s+7}$$

Gap 4

Nyquist Plot: Construction for $q = 0$

Transfer Function

$$G(s) = K\frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$$

Limits

- $\omega \to \infty$: $G(j\omega) \approx K\frac{(j\omega)^m}{(j\omega)^n} = \frac{K}{(j\omega)^{n-m}} = \begin{cases} K & \text{if } m-n=0 \\ 0 & \text{otherwise} \end{cases}$
- $\omega \to 0$: $G(j0) = K \cdot \frac{(-z_1)\cdots(-z_m)}{(-p_1)\cdots(-p_n)}$

Result

- NP approaches point $\lim_{\omega\to\infty} \frac{K}{\omega^{n-m}}$ from phase $-\frac{\pi}{2}(n-m) + \angle(K)$
- NP starts ($\omega = 0$) at $K \cdot \frac{(-z_1)\cdots(-z_m)}{(-p_1)\cdots(-p_n)}$
- NP is symmetric to real axis: $G(j(-\omega)) = G(j\omega)^\star$

Nyquist Plot: Example $G(s) = \frac{s+7}{s+100}$

Computation

Gap 5

Nyquist Plot: Example $G(s) = \frac{s+7}{s+100}$

Computation

Gap 6

Nyquist Plot: Construction for $q \neq 0$

Transfer Function

$$G(s) = K\frac{(s-z_1)\cdots(s-z_m)}{s^q\,(s-p_{g+1})\cdots(s-p_n)}$$

Limits

- $\omega \to \infty$: $G(j\omega) \approx K\frac{(j\omega)^m}{(j\omega)^n} = \frac{K}{(j\omega)^{n-m}} = \begin{cases} K & \text{if } m-n=0 \\ 0 & \text{otherwise} \end{cases}$
- $\omega \to 0$: $G(j\omega) \approx K \cdot \frac{(-z_1)\cdots(-z_m)}{(-p_{q+1})\cdots(-p_n)} \cdot \frac{1}{(j\omega)^q} = K_r \cdot \frac{1}{(j\omega)^q}$

Result

- NP approaches point $\lim_{\omega\to\infty} \frac{K}{\omega^{n-m}}$ from phase $-\frac{\pi}{2}(n-m) + \angle(K)$
- NP starts ($\omega \to 0$) from $|G(j\omega)| = \infty$ with phase $-q\,\frac{\pi}{2} + \angle(K_r)$
- NP is symmetric to real axis: $G(j(-\omega)) = G(j\omega)^\star$

Nyquist Plot: Construction

Small Semi-Circles for poles at zero: $s = r \cdot e^{j\varphi}$, $-\pi/2 \le \varphi \le \pi/2$

$$G(r \cdot e^{j\varphi}) = K \cdot \frac{(r \cdot e^{j\varphi} - z_1)\cdots(r \cdot e^{j\varphi} - z_m)}{(r \cdot e^{j\varphi} - p_{q+1})\cdots(r \cdot e^{j\varphi} - p_n)} \cdot \frac{1}{r^q e^{jq\varphi}}$$
$$\approx K \cdot \frac{(-z_1)\cdots(-z_m)}{(-p_{q+1})\cdots(-p_n)} \cdot \frac{1}{r^q} \cdot e^{-jq\varphi} = K_r \cdot \frac{1}{r^q} \cdot e^{-jq\varphi}$$

Result

- In Nyquist Plot: circular curve with large radius that encircles the origin $q/2$ times
- Curve starts at $\angle(K_r) + q\,\pi/2$ and ends at $\angle(K_r) - q\,\pi/2$
- Direction is clockwise

Nyquist Plot: Example $G(s) = \frac{1}{s\,(s+1)}$

Computation

Gap 7

Nyquist Plot: Example $G(s) = \frac{1}{s\,(s+1)}$

Computation

Gap 8

Nyquist Plot: Example $G(s) = \frac{s+4}{(s^2+2\,s+5)\,(s+10)\,(s+1)}$

Computation

Gap 9

Nyquist Plot: Example $G(s) = \frac{s+4}{(s^2+2\,s+5)\,(s+10)\,(s+1)}$

Computation

Gap 10