ECE 388 – Automatic Control

Stability Analysis: Nyquist Plot and Bode Plot

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Compulsory Course in Electronic and Communication Engineering
Credits (2/2/3)

Course Webpage: http://ECE388.cankaya.edu.tr

Nyquist Stability Criterion: Reminder

Basic Feedback Control Loop

Gap 1

- The feedback loop is internally stable if all zeros of $1 + C(s)\,G(s)$ lie in the OLHP
  - $\Rightarrow$ Direct computation of the zeros of $1 + C(s)\,G(s)$
  - $\Rightarrow$ Analytical verification using Routh-Hurwitz test
  - $\Rightarrow$ Graphical analysis using root locus plot
  - $\Rightarrow$ This week: Graphical verification using Bode plot and Nyquist plot

Nyquist Stability Criterion: Result

Nyquist Criterion

- Consider the basic feedback loop
- We call $G_o(s) = C(s)G(s)$ the open-loop transfer function

Gap 2

- Call P the number of poles of G_o in the open right-half plane
- Call N the number of times the Nyquist plot of G_o encircles the point $(-1, 0)$ in clockwise direction
  $\Rightarrow$ The closed loop is internally stable if and only if $N + P = 0$

Nyquist Stability Criterion: Example

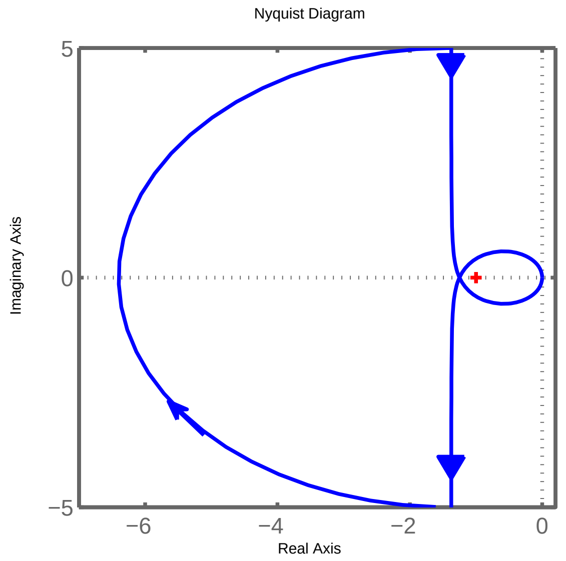
Example

Gap 3

Nyquist Stability Criterion: Example

Instable Open Loop

- $G_o(s) = \dfrac{5s+2}{s(s-4)}$



Computation

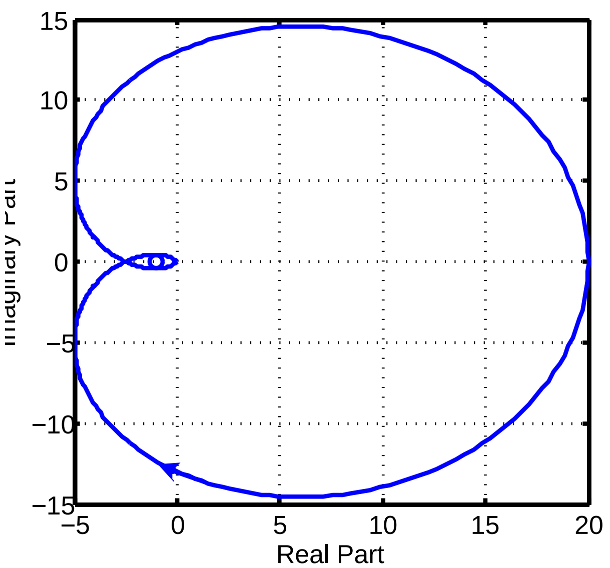
Gap 4

Nyquist Stability Criterion Stability Margins Bode Plot Design Bode Plot Performance

Nyquist Stability Criterion: Example

Stable Open Loop

- $G_o(s) = \dfrac{20}{(1+s)^3}$



Computation

Gap 5

Stability Margins: Gain Margin

Assumption

- Open loop transfer function $G_o(s)$ without poles in the open right half plane

Gain Margin

- Multiplication of G_o with constant K_g leads to instable closed loop
- Phase crossover frequency ω_p such that $\angle G_o(\omega_p) = -\pi$
  $\rightarrow$ Gain margin K_g describes degree of stability with respect to gain changes

Illustration

Gap 6

Stability Margins: Phase Margin

Assumption

- Open loop transfer function $G_o(s)$ without poles in the open right half plane

Phase Margin

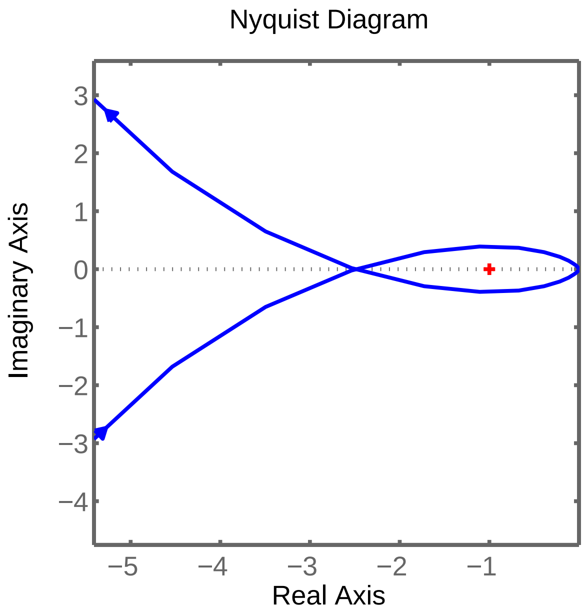
- Multiplication of G_o with $e^{-j\Phi_m}$ (phase shift of Φ_m) leads to instable closed loop
- Gain crossover frequency ω_g such that $|G_o(\omega_g)| = 1$
  $\rightarrow$ Phase margin Φ_m describes degree of stability with respect to phase shift

Illustration

Gap 7

Stability Margins: Example

Example: $G_o(s) = \dfrac{0.005\,(1+s)}{s\,(1+1000\,s)^2}$



Computation

Gap 8

Stability Margins: Bode Plot

Relation between Nyquist Plot and Bode Plot

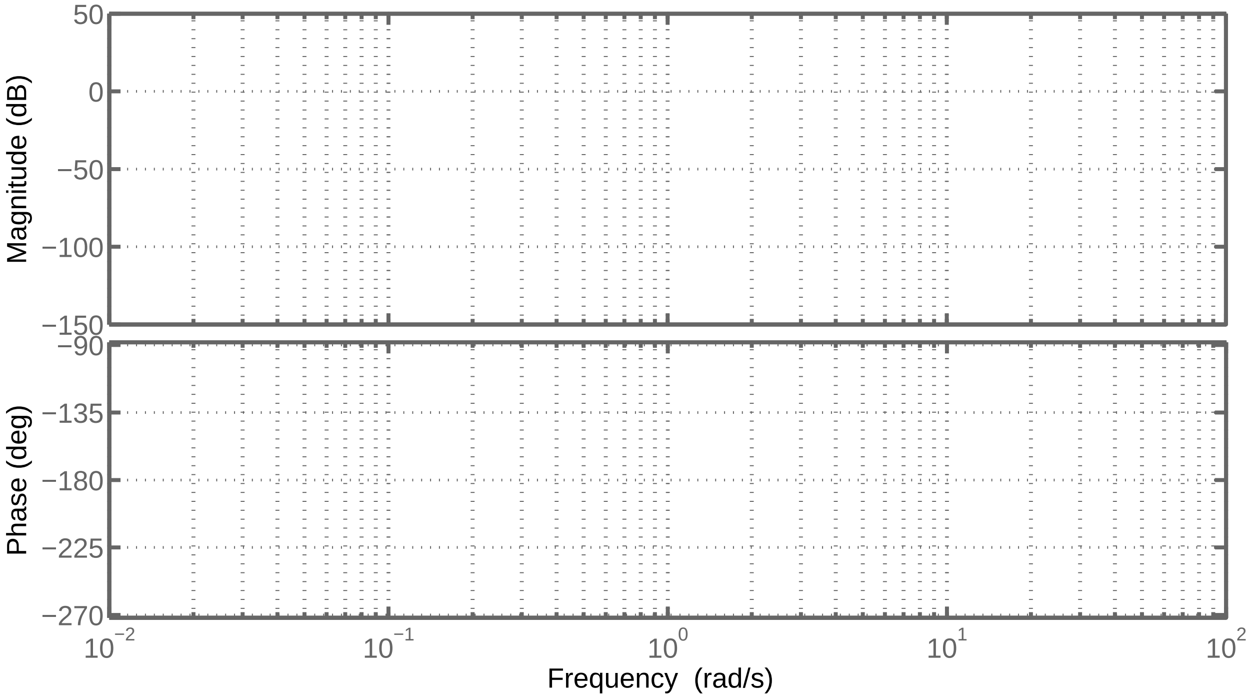
- Magnitude plot: distance of Nyquist curve from origin for each $s = j\omega$
- Phase plot: phase angle of Nyquist curve for each $s = j\omega$
- Gain crossover frequency ω_G: intersection of magnitude plot with 0-dB line
- Phase crossover frequency ω_P: intersection of phase plot with $-180°$

Stability Condition

- Assumption: $G_o(s)$ has no poles in the ORHP
- Condition: Closed loop is internally stable if
  - Phase margin $\Phi_m > 0$
  - Phase between $0 < \omega < \omega_G$ is larger than $-540°$

Stability Margins: Example

Bode Plot



Stability Margins: Example

Nyquist Plot

Gap 9

Bode Plot Performance: Second-Order Lag

Approximated Complimentary Sensitivity

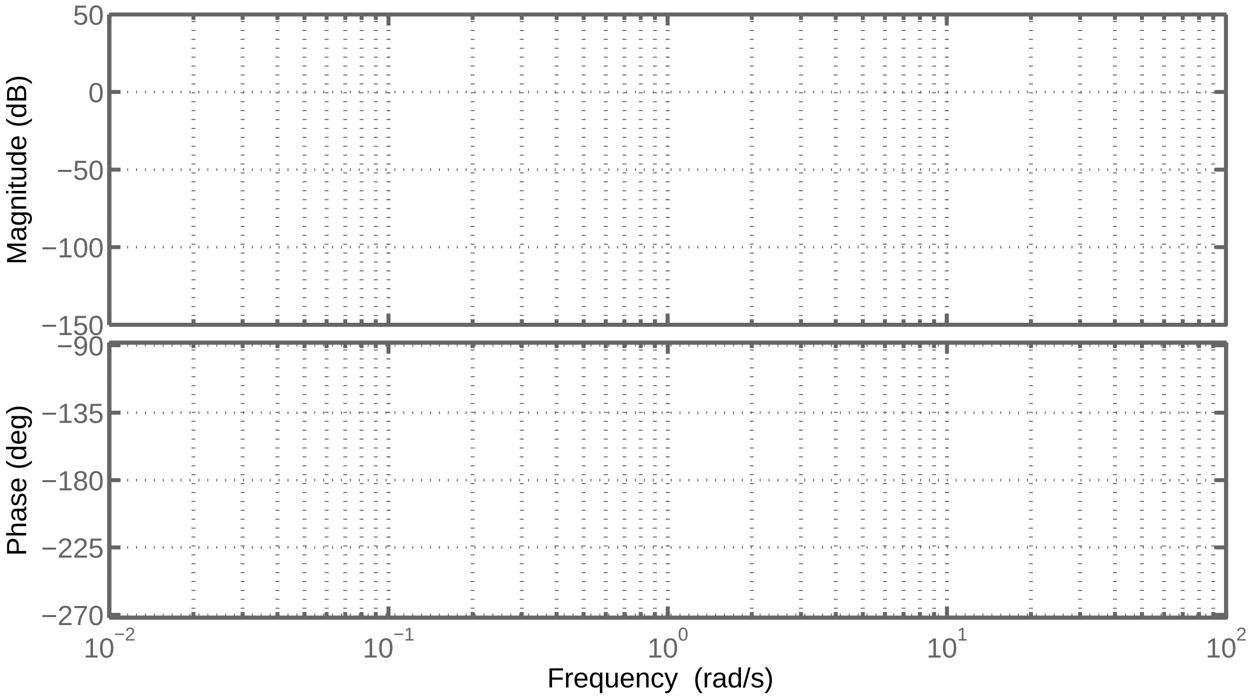
$$T(s) = \frac{K\,\omega_n^2}{\omega_n^2 + 2\,D\,\omega_n\,s + s^2}$$

Performance Parameters

- DC Gain: $K_{DC} = K$
  $\Rightarrow$ Determines final value of the unit step response
- Phase margin: $\Phi_m = \arctan(\frac{2\,D}{\sqrt{-2\,D^2 + \sqrt{1 + 4\,D^4}}})$
  $\Rightarrow$ Determines amplitude of oscillations and relative stability
- Cut-off frequency: $\omega_c = \frac{1}{T}\sqrt{1 - 2\,D^2 + \sqrt{(1 - 2\,D^2)^2 + 1}}$
  $\Rightarrow$ Frequency, where $|T(j\omega)|_{dB}$ is $3\,dB$ below its maximum value
  $\Rightarrow$ Determines speed of response

Bode Plot Performance: Example

Bode Plot of $T(s)$



Bode Plot Performance: Specification

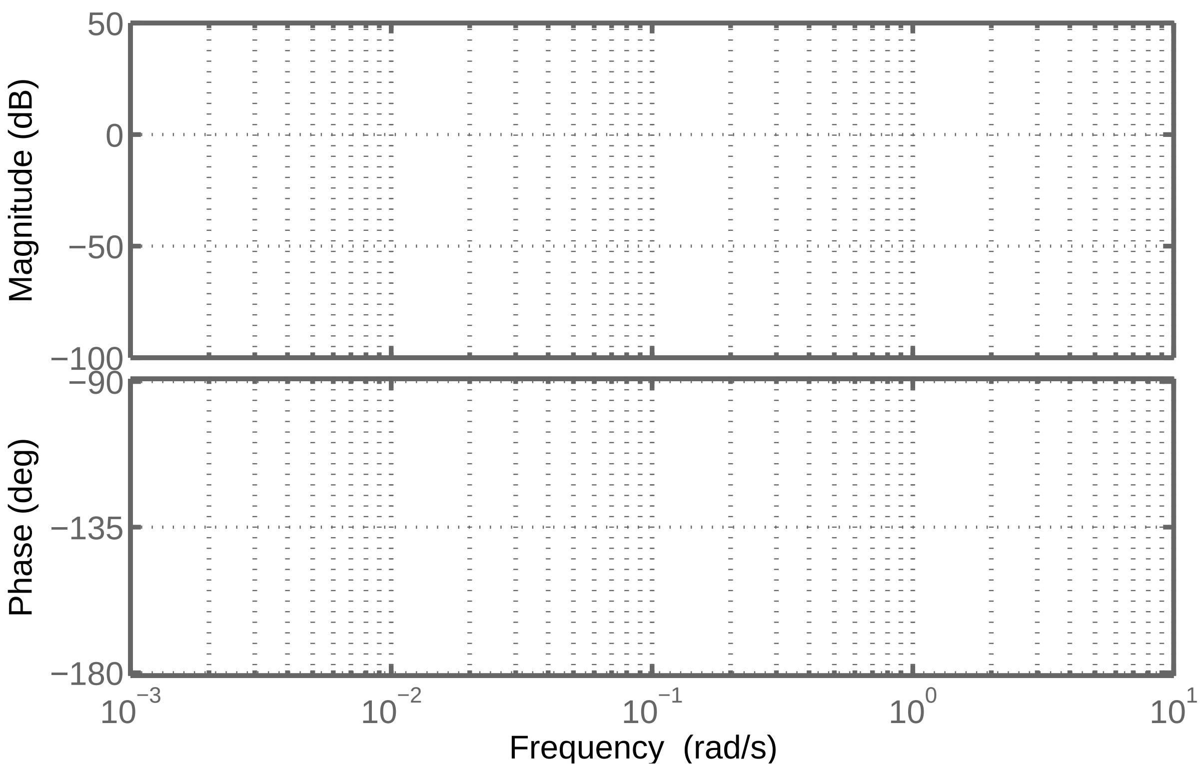
Desired Performance

- DC Gain: $K \approx 1$

Gap 10

- Open-loop transfer function: $G_o(s) = \dfrac{\omega_n}{2\,D} \dfrac{1}{s\,(1 + \frac{1}{2\,D\,\omega_n}\,s)}$
- Sufficient relative stability and small overshoot:
  $\Rightarrow$ Appropriate choice of D. Generally, $D > 0.4$
  $\Rightarrow$ Slope of $|G_o(j\omega)|_{dB} \approx 20\,dB/$decade around ω_c
- Sufficient speed of response:
  $\Rightarrow$ Appropriate choice of cut-off frequency ω_c

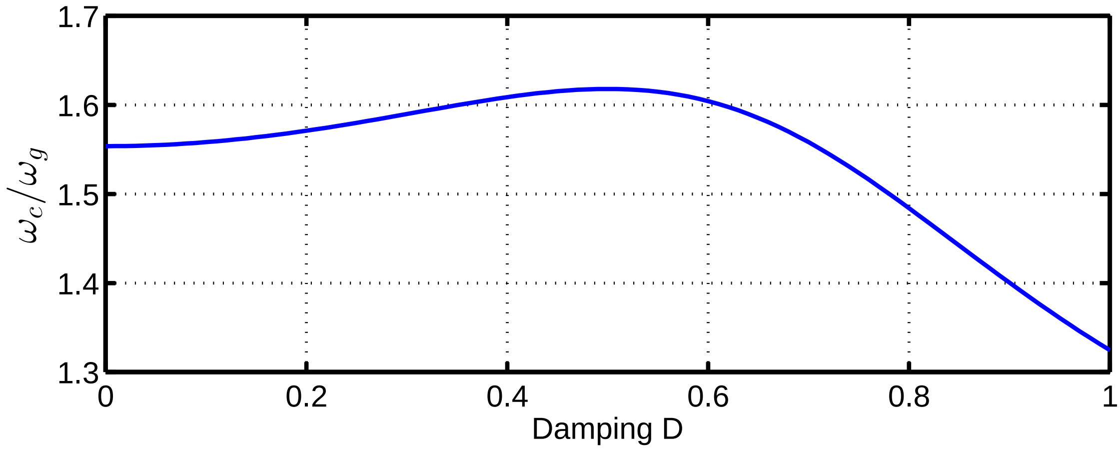
Frequency Response: Open-loop Bode Plot



Closed-loop Requirements: Open-loop Transfer Function

Parameters in Bode Diagram

- Gain crossover frequency: $\omega_g = \frac{1}{T}\sqrt{-2\,D^2 + \sqrt{1 + 4\,D^4}}$
  $\Rightarrow$ It holds that $\omega_g \approx \omega_c$



- Phase margin: Φ_m
  $\Rightarrow$ Compute from stability requirement. Usually, $\Phi_m > 40^\circ$

Closed-loop Requirements: Basic Control Design

Desired Open Loop Transfer Function

- Determine the closed-loop requirements
  $\Rightarrow \omega_c$ and Φ_m
- Determine the desired open-loop transfer function $G_o(s)$
  $\Rightarrow \omega_g \approx \omega_c$ and Φ_m
  $\Rightarrow$ Slope of $|G_o(j\omega)|_{dB} \approx 20\,dB/$decade around ω_g

Adjust Real Open Loop Transfer Function

- Determine the Bode plot of the plant transfer function $G(s)$
  $\Rightarrow$ Fixed part of the control system
- Design the controller transfer function $C(s)$.
  $\Rightarrow$ Choose the controller poles, zeros and gain such that Bode plot of $C(s)\,G(s)$ approximately matches the desired $G_o(s)$ around the frequency ω_g