ECE 388 – Automatic Control

Lead Compensator and PID Control

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Compulsory Course in Electronic and Communication Engineering
Credits (2/2/3)

Course Webpage: http://ECE388.cankaya.edu.tr

Lead Compensator: Usage

Goal

- Reshape frequency response curve to give additional phase lead in order to increase the phase margin

Starting Point

- Plant transfer function $G(s)$
- Lead compensator transfer function $C(s) = K_\alpha \dfrac{1 + T\,s}{1 + \alpha\,T\,s}$
- Desired phase margin Φ_m
- Steady-state error e_∞

Task

- Determine the parameters K_α, α and T

Lead Compensator: Transfer Function

Time-constant Representation

$$C(s) = K_\alpha \, \frac{1 + T\,s}{1 + \alpha\,T\,s}$$

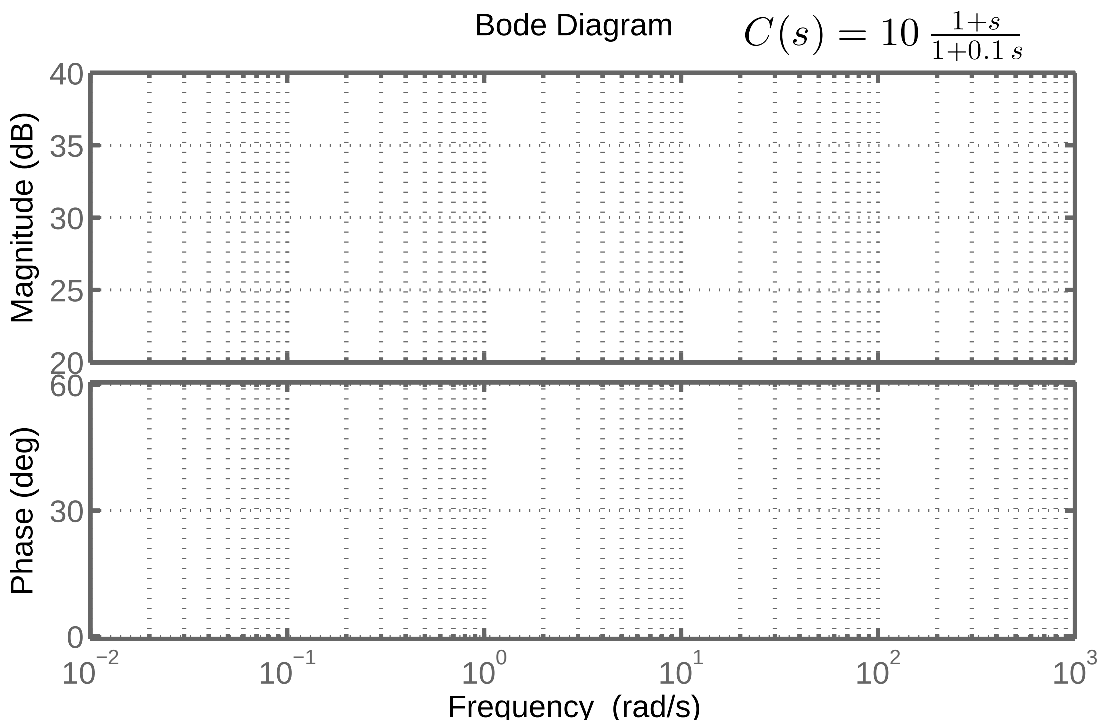
Explanation

- Attenuation factor $0 < \alpha < 1$
- Gain K_α
- Pole at $s = -\frac{1}{\alpha\,T}$
- Zero at $s = -\frac{1}{T}$

Remarks

- Phase increase (lead) up to a maximum value of $\sin(\varphi_\alpha) = \frac{1-\alpha}{1+\alpha}$
- Frequency of maximum phase lead at $\omega_\alpha = \frac{1}{\sqrt{\alpha}\,T}$
- Magnitude at ω_α is $|C(j\omega_\alpha)| = \frac{1}{\sqrt{\alpha}} > 1$

Lead Compensator: Bode Plot



Lead Compensator: Procedure

1. Determine the gain K_α to achieve the static error specification
2. Draw a Bode plot of $K_\alpha\, G(j\omega)$ and determine the phase margin Φ'_m
3. Determine the required lead angle $\varphi_\alpha = \Phi_m - \Phi'_m + 10^\circ$
   $\Rightarrow \alpha = \dfrac{1-\sin(\varphi_\alpha)}{1+\sin(\varphi_\alpha)}$
4. Choose the gain crossover frequency ω_g such that
   $|K_\alpha\, G(j\omega_g)|_{dB} = -20\log(\frac{1}{\sqrt{\alpha}})$
   $\Rightarrow$ Choose $\omega_\alpha = \omega_g$
5. Evaluate $\omega_\alpha = \frac{1}{\sqrt{\alpha}\,T}$
   $\Rightarrow T = \dfrac{1}{\omega_\alpha\sqrt{\alpha}}$
6. Verify if the design fulfills the specified requirements. Go back to step 3. if the requirements are not fulfilled

Lead Compensator: Example

Computation

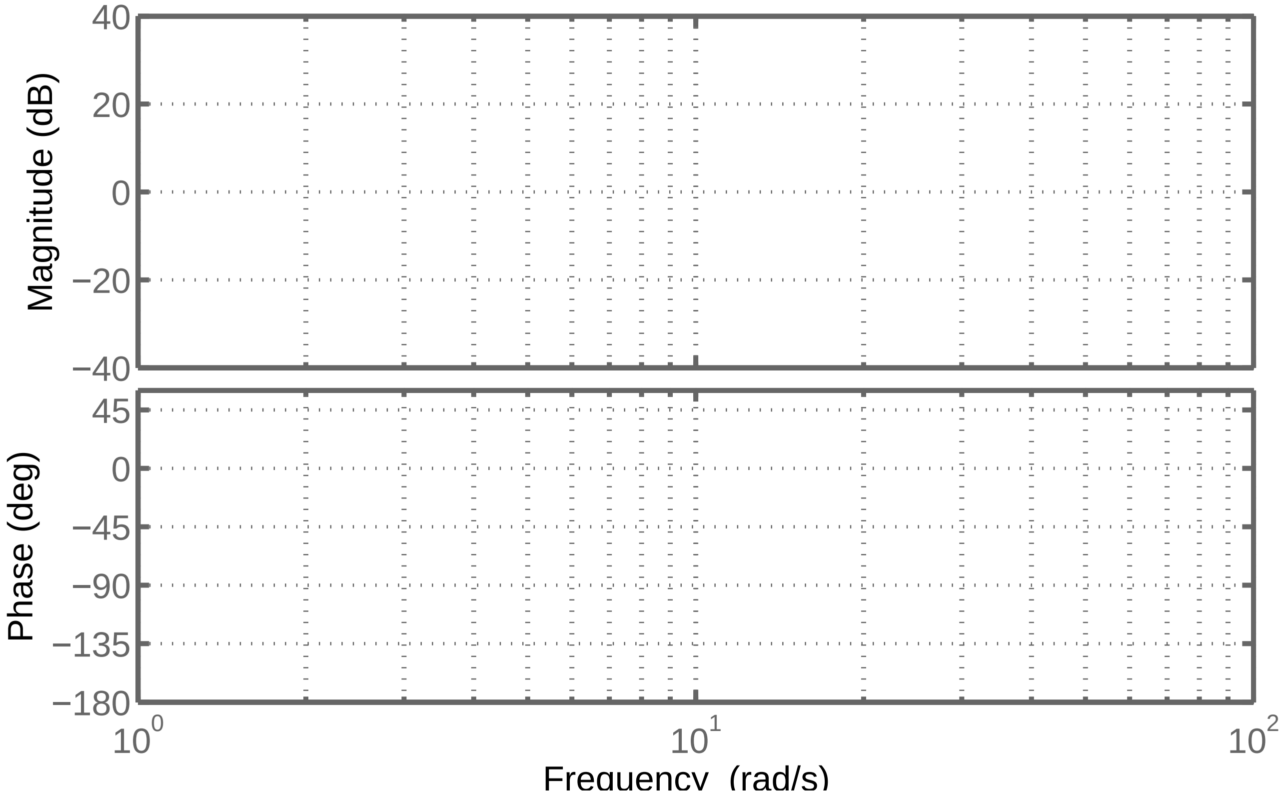
Gap 1

Lead Compensator: Example

Computation

Gap 2

Lead Compensator: Example



PID Controller: Characteristics

Gap 3

Ordinary Differential Equation (ODE)

$$u = K_p \cdot (e + \frac{1}{T_I}\int e + T_D\dot{e})$$

Transfer Function (TF)

$$U(s) = K_p \cdot (E(s) + \frac{1}{T_I\, s}E(s) + T_D\, s\, E(s) = K_p\,(1 + \frac{1}{T_I\, s} + T_D\, s)\, E(s)$$

PID Controller: Parameters

Proportional Action: $K_p \cdot e$

- Depends on instantaneous value of error
- Can control any stable plant but usually with low performance

Integral Action: $\frac{K_p}{T_I} \cdot \int e$

- Realizes memory due to dependency on accumulated error
- Enforces steady state error of $\lim_{t\to\infty} e(t) = 0$

Derivative Action: $K_p T_D \cdot \dot{e}$

- Captures trend of the error due to dependency on rate of change of e
- Susceptible to amplification of high-frequency disturbances/noise

PID Controller: Parameters

Illustration

Gap 4

PID Controller: Special Cases

P-Controller

$$C(s) = K_p$$

PI-Controller

$$C(s) = K_p\,(1 + \frac{1}{T_I\,s})$$

PD-Controller

$$C(s) = K_p\,(1 + T_D\,s)$$

Design Task

- Determine the most suitable controller type and the controller parameters K_p, T_I and T_D in order to fulfill given performance specifications

Ziegler-Nichols: Oscillation Method

Assumption

- Stable, non-oscillatory plant: $G(s) = K \frac{e^{-s\tau}}{(1+sT_1)\cdots(1+sT_n)}$
  excluding first-order/second-order lag
- Note: plant is not modeled!

Practical Experiment

- Start with $K_p = 0$ and increase K_p gradually until y oscillates
  $\Rightarrow$ Critical gain K_{crit}
- Note oscillation period T_{crit}

Control Loop with P-control

Gap 5

Ziegler-Nichols: Oscillation Method

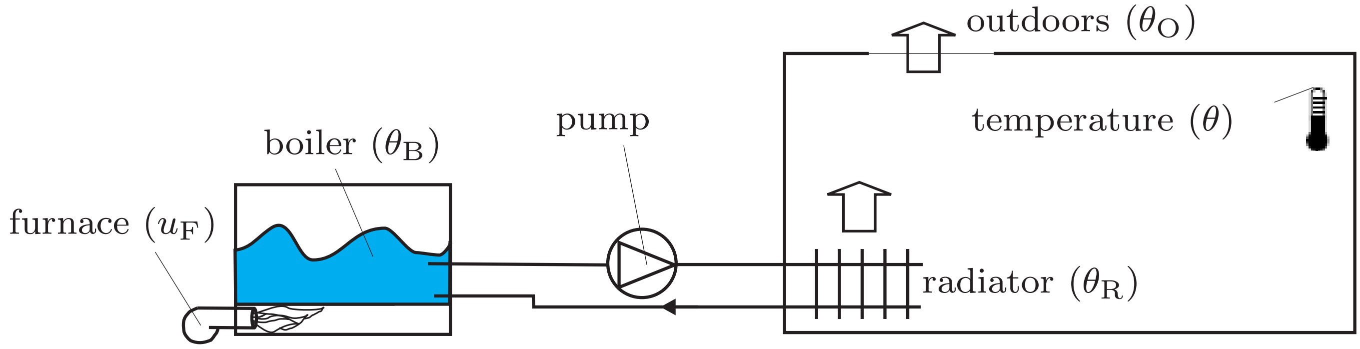
PID-controller Parameters

Controller	K_p	T_I	T_D
P-	$0.5K_{\text{crit}}$	∞	0
PI-	$0.45K_{\text{crit}}$	$0.85T_{\text{crit}}$	0
PID-	$0.6K_{\text{crit}}$	$0.5T_{\text{crit}}$	$0.12T_{\text{crit}}$

Results

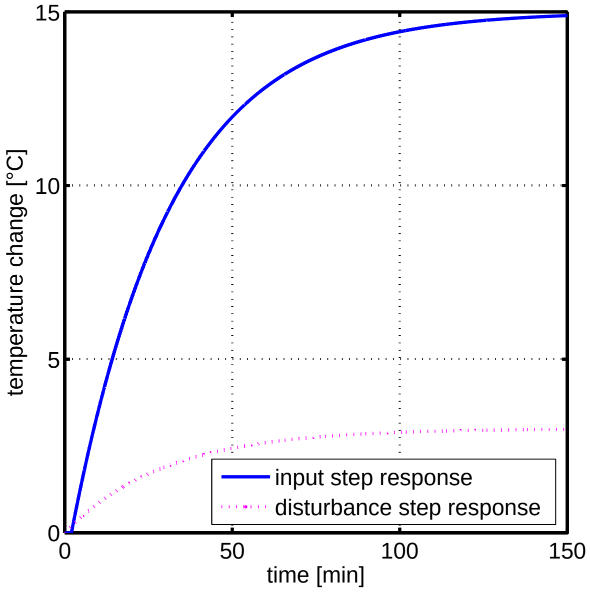
- Stable closed loop
- Addresses both reference tracking and disturbance rejection

Example (temperature control)

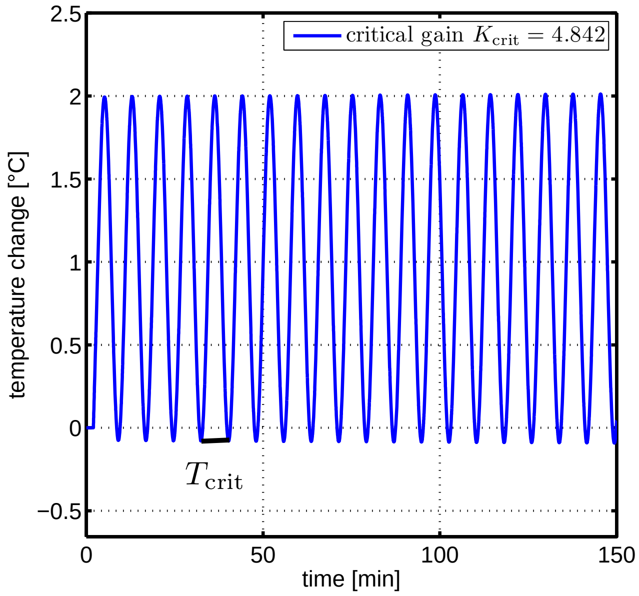


Ziegler-Nichols: Oscillation Method

Uncontrolled Plant Step Response



Oscillation Experiment



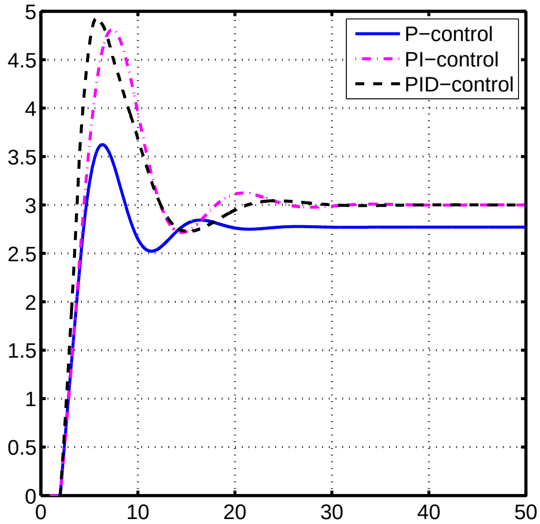
Ziegler-Nichols: Example

Computation

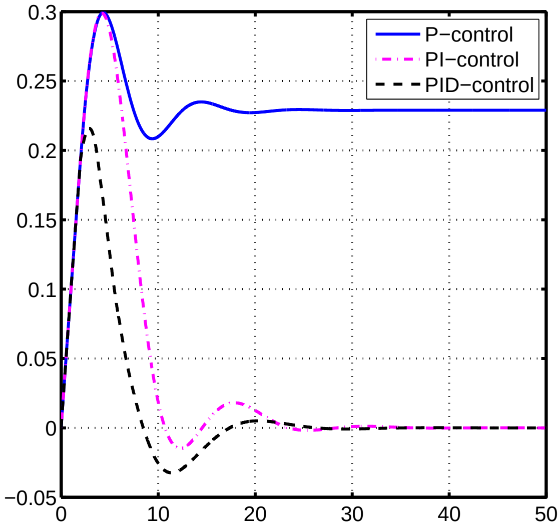
Gap 6

Ziegler-Nichols: Oscillation Method

Reference Step Response



Disturbance Step Response



$\Rightarrow$ Nonzero static position error with P-control

$\Rightarrow$ Larger overshoot for PI and PID control due to plant delay

$\Rightarrow$ Similar dynamics for reference tracking and disturbance rejection

Ziegler-Nichols: Reaction Curve Method

Assumption

- Stable, non-oscillatory plant: $G(s) = K \dfrac{e^{-s\tau}}{(1+sT_1)\cdots(1+sT_n)}$

  excluding $G(s) = \dfrac{K}{1+sT_1}$
- Note: plant is not modeled!

Practical Experiment

- Approach desired set-point
- Apply "small" step input
- Record plant output: process reaction curve

Step Response in Open Loop

Gap 7

Ziegler-Nichols: Reaction Curve Method

Characteristic Plant Parameters

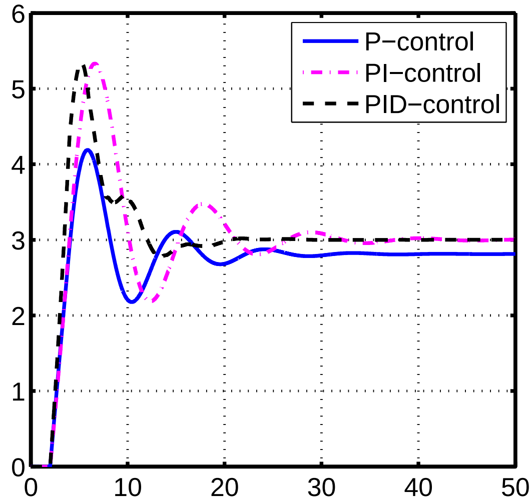
Gap 8

PID-controller Parameters

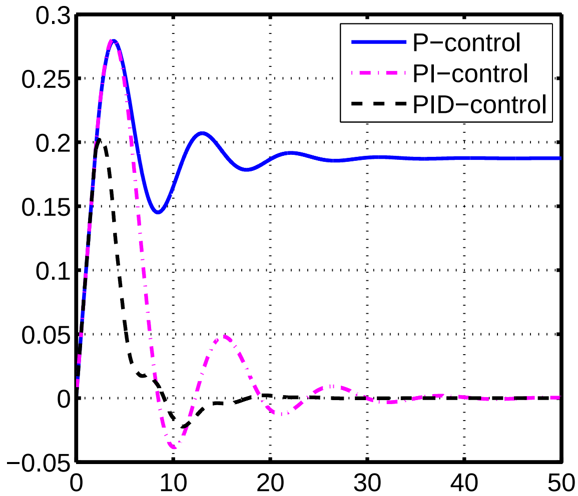
Controller	K_p	T_I	T_D
P-	$1/K \cdot T/\tau$	∞	0
PI-	$0.9/K \cdot T/\tau$	3.33τ	0
PID-	$1.2/K \cdot T/\tau$	2τ	0.5τ

Ziegler-Nichols: Oscillation Method

Reference Step Response



Disturbance Step Response



$\Rightarrow$ Similar behavior to Ziegler-Nichols Oscillation Method