ECE 388 – Automatic Control

Feedback Loop – Performance Specifications

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Compulsory Course in Electronic and Communication Engineering
Credits (2/2/3)

Course Webpage: http://ECE388.cankaya.edu.tr

Feedback Control Example: DC Motor

Block Diagram

Gap 1

Transfer Functions

$$G_1(s) = \frac{25}{s+5000}$$

$$G_2(s) = G_d(s) = \frac{32 \cdot 10^4}{(s+85)}$$

$$G(s) = G_1(s)\, G_2(s) = \frac{8 \cdot 10^6}{(s+85)\,(s+5000)}$$

Feedback Control Example: DC Motor

Task

- Keep the rotational velocity constant at 20 $\mathrm{rad/sec}$ even in case of disturbances

Possible Strategy

- Proportional correction of output errors

Gap 2

Feedback Control Example: DC Motor

Proportional Control

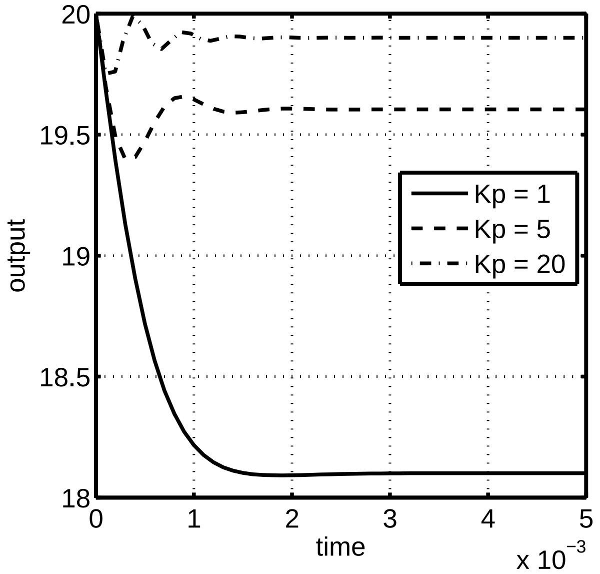
Gap 3

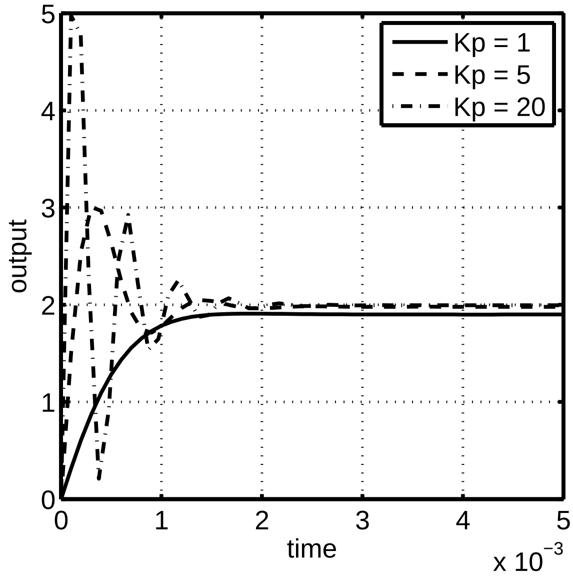
Remarks

- K_p can be considered as controller transfer function
- Different choices of K_p will lead to different behavior of the closed loop
- Larger values of K_p will lead to larger control input signals

Feedback Control Example: DC Motor

Disturbance Step $T_L = 10^{-2}$ Nm





$\Rightarrow$ Stable feedback loop for all choices of K_p

$\Rightarrow$ Steady-state error decreases with larger value of K_p

Feedback Control Example: Magnetic Suspension

Block Diagram

Gap 4

Transfer Functions

$$G_1(s) = 1 \qquad G_2(s) = G_d(s) = \frac{0.01}{(-s^2 + 1000)}$$

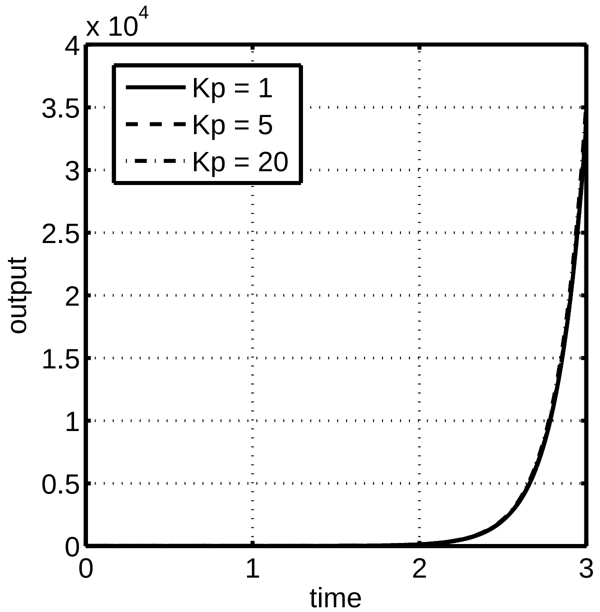
$$G(s) = G_1(s)\, G_2(s) = \frac{0.01}{(-s^2 + 1000)}$$

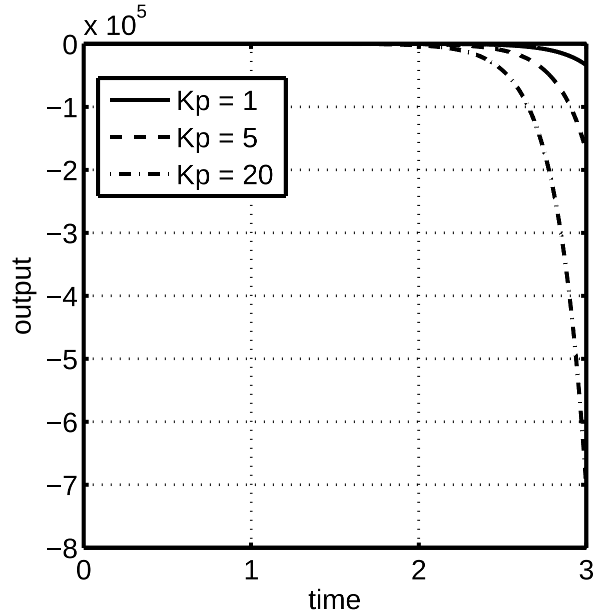
Task

- Keep the position constant

Feedback Control Example: Magnetic Suspension

Disturbance Step $F_z = 10^{-2}$ N





$\Rightarrow$ Instable feedback loop for all choices of K_p

$\Rightarrow$ Proportional control seems not suitable for the magnetic suspension

Basic Feedback Control Loop: Description

Block Diagram

Gap 5

Description

$G(s)$	r	d_i	d
plant transfer function	reference signal	input disturbance signal	output disturbance signal
$C(s)$	e	u	y
controller transfer function	error signal	control input signal	output signal

Basic Feedback Control Loop: Sensitivities

Complementary Sensitivity (Reference signal to output signal)

$$T(s) = \frac{Y(s)}{R(s)} = \frac{C(s)\,G(s)}{1 + C(s)\,G(s)}$$

Sensitivity (Output disturbance signal to output signal)

$$S(s) = \frac{Y(s)}{D(s)} = \frac{1}{1 + C(s)\,G(s)}$$

Input Sensitivity (Input disturbance signal to output signal)

$$S_i(s) = \frac{Y(s)}{D_i(s)} = \frac{G(s)}{1 + C(s)\,G(s)}$$

Control Sensitivity (Reference signal to control input signal)

$$S_u(s) = \frac{U(s)}{R(s)} = \frac{C(s)}{1 + C(s)\,G(s)}$$

Basic Feedback Control Loop: Sensitivities

Computation

Gap 6

Basic Feedback Control Loop: Internal Stability

Definition

The basic feedback loop is **internally stable** *if and only if all four sensitivities are stable transfer functions*

$\Rightarrow$ All signals in the basic feedback control loop are bounded if the input signals r, d, d_i are bounded

General Stability Test

- The feedback loop is internally stable if all zeros of $1 + G(s)C(s)$ lie in the OLHP

General Statements

- If $G(s)$ and $C(s)$ are stable, then stability of $T(s)$ is sufficient for internal stability of the basic feedback control loop
- The feedback loop is instable if $C(s)$ cancels an instable pole or a non-minimum phase zero of $G(s)$ (see next slide)

Basic Feedback Control Loop: Example

Computation

Gap 7

Steady-State Error: Description

Open-loop Transfer Function Type

$$G_o(s) = C(s)\,G(s) = \frac{B(s)}{s^q\,A'(s)}$$

$\Rightarrow$ We say $G_o(s)$ is of type q

Explanation

- Steady-state error: Difference between reference and output signal in the steady state for step inputs
- Note: steady state error is only bounded if the feedback loop is internally stable

Computation: Final Value Theorem

$$\lim_{t\to\infty}(r(t) - y(t)) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} E(s)\,s = \lim_{s\to 0} S(s)\,\frac{1}{s}\,s = \lim_{s\to 0} S(s)$$

$\Rightarrow$ Steady state error is computed from $S(s) = \dfrac{1}{1 + C(s)\,G(s)}$

Steady-State Error: Example

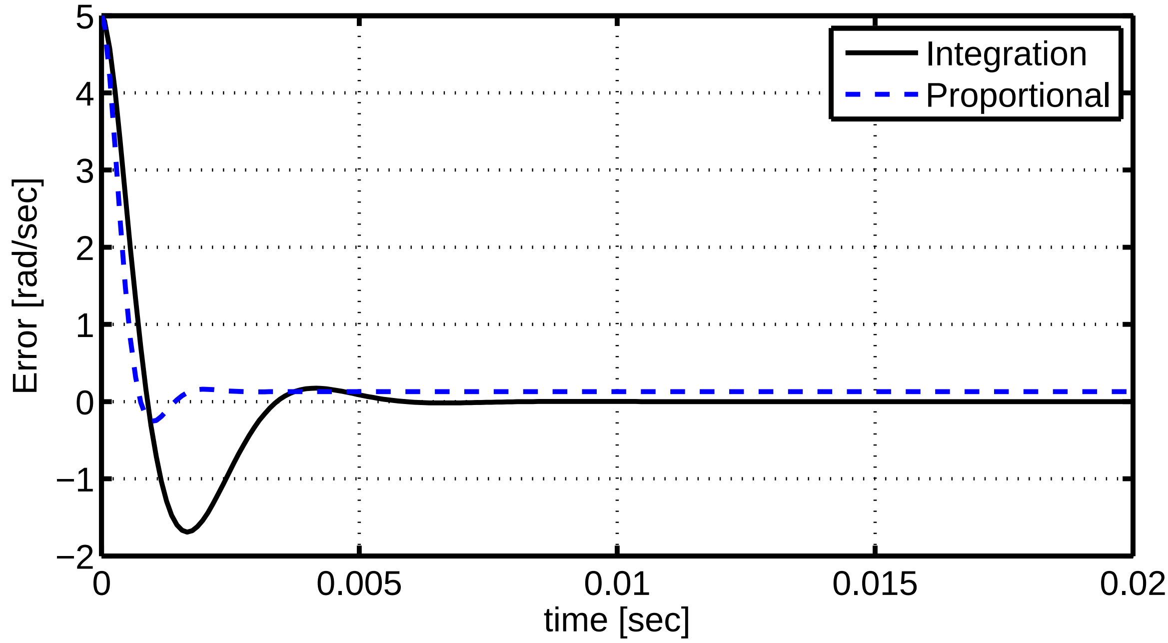
Computation

Gap 8

$\Rightarrow$ Proportional control leads to non-zero steady state error
$\Rightarrow$ Controller with integrator achieves zero steady state error

Steady-State Error: Simulation

DC Motor



Performance Specifications: Step Response Characteristics

Performance Specifications

- Describe desired behavior of a control system based on step response
- Define performance metrics that can be used in practice

Illustration

Gap 9

Performance Specifications: Step Response Characteristics

Steady State Value

- Output in steady state: $y_\infty = \lim_{t\to\infty} y(t)$

Rise Time

- Quantifies speed of response: first time t_r such that $y(t_r) = 0.95 y_\infty$

(Percent) Overshoot

- Quantifies damping of response: $M_p = \max_{t\in\mathbb{R}} \frac{y(t) - y_\infty}{y_\infty}$

Peak Time

- Time until first peak of overshoot is reached: t_p

Settling Time

- Quantifies how long it takes until response stays around the final value (for example 2% or 5%): t_s

Performance Specifications: Example

First-order Lag

$$Y(s) = \frac{K}{1 + s\,T}\,\frac{1}{s} \;\bullet\!\!-\!\!\circ\; y(t) = \sigma(t)(1 - e^{-t/T})$$

Computation

Gap 10

Performance Specifications: Second-Order Lag

Transfer Function

$$T(s) = \frac{K\,\omega_n^2}{\omega_n^2 + 2\,D\,\omega_n\,s + s^2}$$

Characteristics

- Rise time: $t_r = \dfrac{1}{\omega_n\sqrt{1-D^2}}\ \arctan(\dfrac{\sqrt{1-D^2}}{D})$
- Peak time: $t_p = \dfrac{\pi}{\omega_n\sqrt{1-D^2}}$
- Overshoot: $M_p = e^{-\frac{D}{\sqrt{1-D^2}}\pi}$
- Settling time: $t_s = \dfrac{3}{\omega_n\,D}$ (5%); $t_s = \dfrac{4}{\omega_n\,D}$ (2%)

Performance Specifications: Example

Computation

Gap 11

Performance Specifications: Relation to Poles

Closed-loop Poles for Second-order Lag

$$p_{1/2} = -\omega_n D \pm \omega_n \sqrt{D^2 - 1}$$

Computation

Gap 12

⇒ Real part of poles determines settling time
⇒ Pole angle determines damping

Performance Specifications: Location in the Complex Plane

General Rule for Second-order Lags

- For given t_s (2%) $\Rightarrow Re(p_{1/2}) < -4/t_s$
- For given $D < 1 \Rightarrow \beta := \pi - |\angle(p_{1/2})| < \arctan \sqrt{1 - D^2}/D$

Illustration

Gap 13