

ECE 388 – Automatic Control

Bode Plot

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Compulsory Course in Electronic and Communication
Engineering
Credits (2/2/3)

Course Webpage: <http://ECE388.cankaya.edu.tr>

TF Representations: Numerator/Denominator

Denominator/Numerator Representation

$$G(s) = \frac{B(s)}{A(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

- m : numerator degree, n denominator degree

Example

Gap 1

TF Representations: Pole-Zero

Pole-Zero Representation

$$G(s) = K \frac{(s - z_1) \cdot (s - z_2) \cdots (s - z_m)}{(s - p_1) \cdot (s - p_2) \cdots (s - p_n)}$$

- z_j : zeros; p_i : poles; K : constant

Example

Gap 2

TF Representations: Time-constant

Time-constant Representation

$$G(s) = K_{DC} \frac{(1 + \tau_1 s)(1 + \tau_2 s) \cdots (1 + 2\delta_f \tau_f s + \tau_f^2 s^2) \cdots}{s^q (1 + T_1 s)(1 + T_2 s) \cdots (1 + 2D_g T_g s + T_g^2 s^2) \cdots}$$

- Time constants for real zeros/poles: τ_1, τ_2, \dots ; T_1, T_2, \dots
- Damping for conjugated complex zeros/poles: δ_f, \dots ; D_g, \dots
- Number of integrators: q

Example

Gap 3

Frequency Response: Basic Idea

Given

- Stable LTI system with transfer function $G(s)$

Goal

- Find system response $y(t)$ for sinusoidal input signal

$$u(t) = \sin(\omega t)$$

Gap 4

Solution

- Consider output computation in the Laplace domain:

$$Y(s) = G(s) U(s)$$

- Sinusoidal input function: $U(s) = \frac{\omega}{s^2 + \omega^2}$

Frequency Response: Result

Result

$$y(t) = |G(j\omega)| \sin(\omega t + \angle(G(j\omega)))$$

Description

- Output signal y oscillates with same frequency ω as input signal u
- Amplification of u by $|G(j\omega)|$
- Phase shift of u by $\angle(G(j\omega))$

Illustration

Gap 5

Bode Plot: Basic Idea

Description

- Given: Transfer function $G(s)$
- Task: Show the frequency response in terms of magnitude $|G(j\omega)|$ and phase shift $\angle(G(j\omega))$

Magnitude Plot

- Frequency axis with logarithmic scale ω [rad/sec]
- Magnitude axis with $20 \log |G(j\omega)|$ [dB]

Phase Plot

- Frequency axis with logarithmic scale ω [rad/sec]
- Phase axis with $\angle G(j\omega) = \arctan\left(\frac{\text{Im}(G(j\omega))}{\text{Re}(G(j\omega))}\right)$ [°]

Bode Plot: Example

Bode Plot Example

