ECE 388 – Automatic Control

Bode Plot

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Compulsory Course in Electronic and Communication Engineering
Credits (2/2/3)

Course Webpage: http://ECE388.cankaya.edu.tr

Bode Plot: Basic Idea

Description

- Given: Transfer function $G(s)$
- Task: Show the frequency response in terms of magnitude $|G(j\omega)|$ and phase shift $\angle(G(j\omega))$

Magnitude Plot

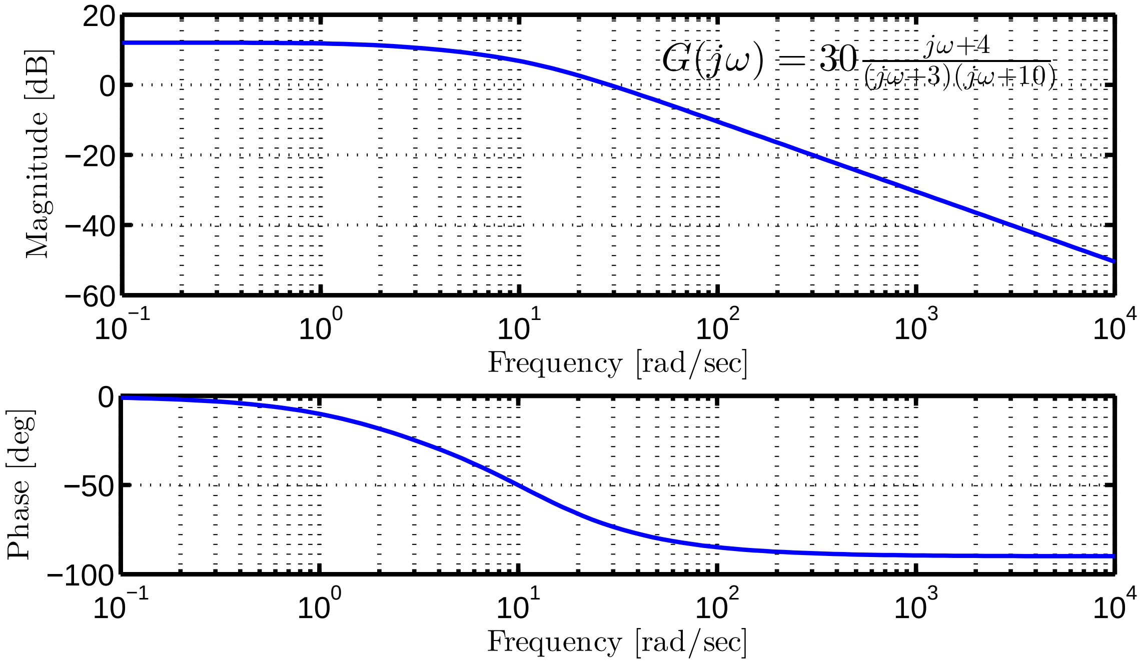
- Frequency axis with logarithmic scale ω [rad/sec]
- Magnitude axis with $20\log|G(j\omega)|$ [dB]

Phase Plot

- Frequency axis with logarithmic scale ω [rad/sec]
- Phase axis with $\angle G(j\omega) = \arctan(\frac{\mathrm{Im}(G(j\omega))}{\mathrm{Re}(G(j\omega))})$ [°]

Bode Plot: Example

Bode Plot Example



Bode Plot: Transfer Function Representation

Time-constant Representation

$$G(s) = K_{\mathrm{DC}}\frac{(1+\tau_1 s)(1+\tau_2 s)\cdots(1+2\delta_f\tau_f s+\tau_f^2 s^2)\cdots}{s^q(1+T_1 s)(1+T_2 s)\cdots(1+2D_g T_g s+T_g^2 s^2)\cdots}$$

- Time constants for real zeros/poles: $\tau_1, \tau_2, \cdots;\ T_1, T_2, \cdots$
- Time constants for conjugated complex zeros/poles: $\tau_f, \cdots;\ T_g, \cdots$
- Damping for conjugated complex zeros/poles: $\delta_f, \cdots;\ D_g, \cdots$
- Multiplicity of pole at zero: q
  $\Rightarrow$ If the transfer function is not given in the time-constant representation, it has to be transformed to this representation

Frequency Response

$$G(s) = K_{\mathrm{DC}}\frac{(1+j\omega\tau_1)(1+j\omega\tau_2)\cdots(1+j\omega 2\delta_f\tau_f+(j\omega)^2\tau_f^2)\cdots}{(j\omega)^q(1+j\omega T_1)(1+j\omega T_2)\cdots(1+j\omega 2D_g T_g+(j\omega)^2 T_g^2)\cdots}$$

Bode Plot: Transfer Function Representation

Example

Gap 1

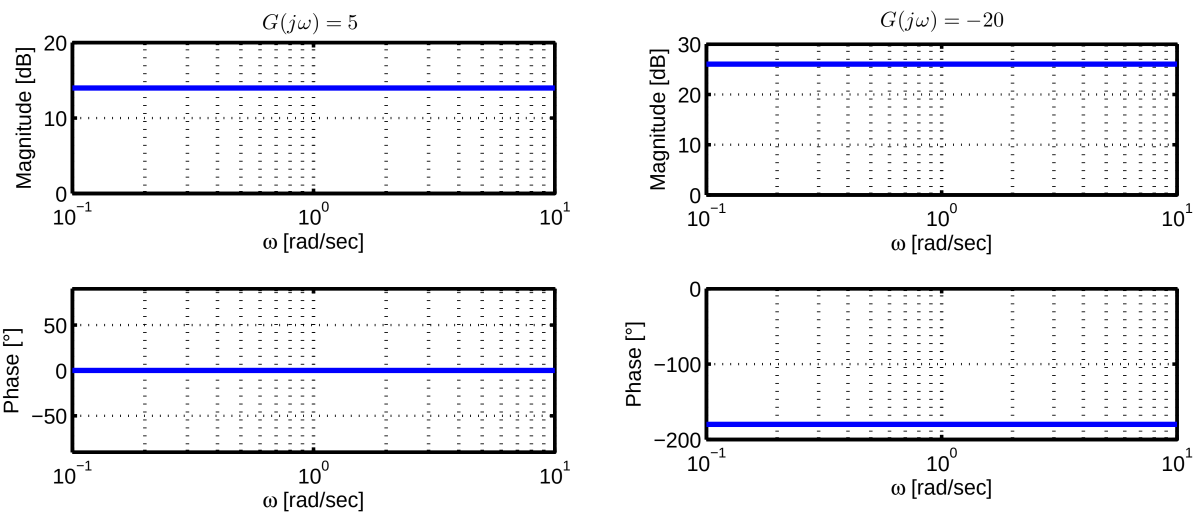
Standard Numerator/Denominator Factors

- K_{DC}
- s
- $1 + T\,s$
- $\omega_n^2 + 2\,D\,\omega_n\,s + s^2$

Bode Plot: Common Examples

DC Gain: $G(j\omega) = K_{DC}$

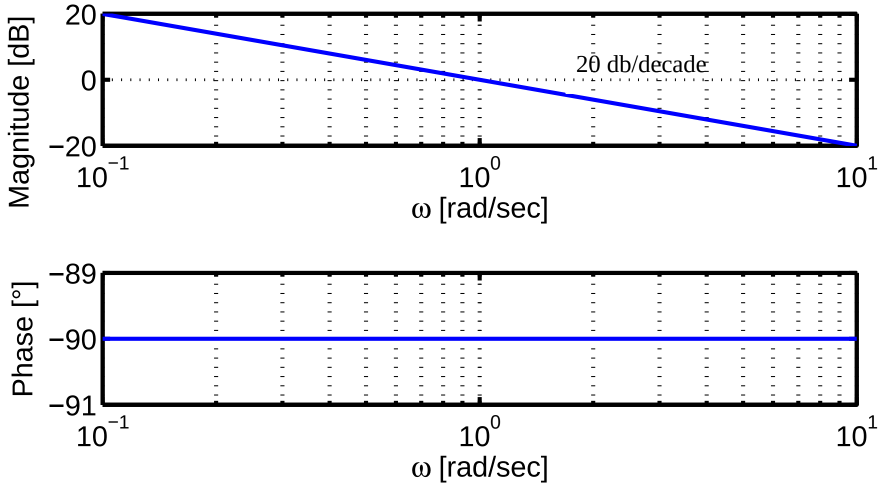
- Magnitude: $|G(j\omega)| = |K_{DC}| \Rightarrow |G(j\omega)|_{dB} = 20 \log |K_{DC}|$
- Phase $\angle(G(j\omega)) = \begin{cases} 0^\circ & \text{if } K_{DC} > 0 \\ 180^\circ & \text{if } K_{DC} < 0 \end{cases}$



Bode Plot: Common Examples

Integrator $G(j\omega) = \dfrac{1}{j\omega}$

- Magnitude: $|G(j\omega)| = \dfrac{1}{\omega} \Rightarrow |G(j\omega)|_{dB} = -20\log\omega$
- Phase: $\angle G(j\omega) = -90^\circ$



Bode Plot: Examples

First-order Lag $G(j\omega) = \dfrac{1}{1 + j\omega T}$

- Magnitude: $|G(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2 T^2}} \approx \begin{cases} 0 & \omega < 1/T \\ -20\log\omega T & \omega > 1/T \end{cases}$

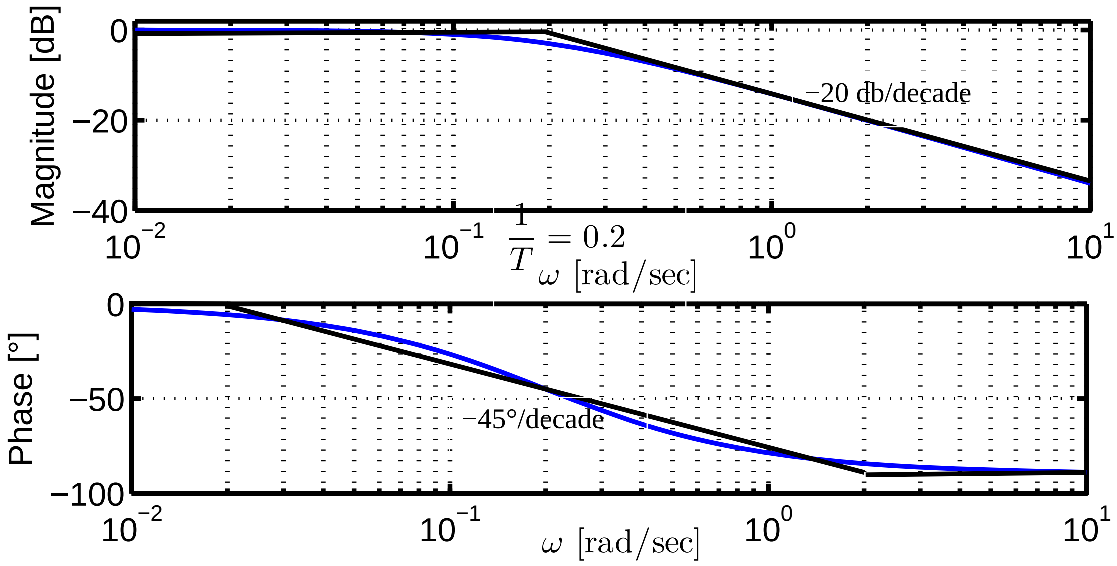
  $\Rightarrow$ Straight-line approximation that bends at $\omega = 1/T$
- Phase: $\angle G(j\omega) = -\arctan\omega T \approx \begin{cases} 0^\circ & \omega < 1/(10\,T) \\ -90^\circ & \omega > 10/T \end{cases}$

$\Rightarrow$ Straight-line approximation that decreases from 0 to -90° between $\omega = 1/(10\,T)$ to $\omega = 10/T$

Gap 2

Bode Plot: First-order Lag

Bode Plot Construction: $G(s) = \dfrac{1}{1 + 5\,s}$



Bode Plot: Second-order Lag

Second-order Lag $G(j\omega) = \dfrac{1}{1 + 2\,D\,j\omega/\omega_n + (j\omega/\omega_n)^2}$

- Magnitude:

  $|G(j\omega)| = \frac{1}{\sqrt{(1-(\omega/\omega_n)^2)^2 + 4D^2(\omega/\omega_n)^2}} \approx \begin{cases} 0 & \omega < \omega_n \\ -40\log\omega/\omega_n & \omega > \omega_n \end{cases}$

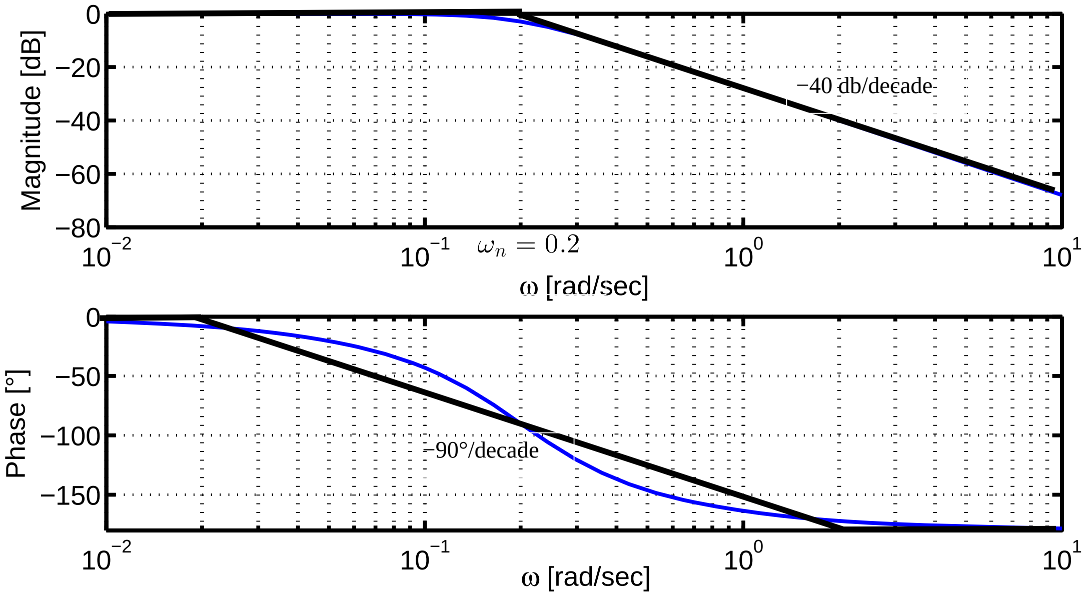
  $\Rightarrow$ Straight-line approximation that bends at $\omega = 1/T$
- Phase: $\angle G(j\omega) = -\arctan\frac{2\,D\,\omega/\omega_n}{1-(\omega/\omega_n)^2} \approx \begin{cases} 0^\circ & \omega << \omega_n/10 \\ -180^\circ & \omega >> 10\,\omega_n \end{cases}$

  $\Rightarrow$ Straight-line approximation that decreases from 0 to -180° between $\omega = \omega_n/10$ and $\omega = 10\,\omega_n$

Gap 3

Bode Plot: Examples

Bode Plot Construction: $G(s) = \dfrac{1}{1 + 10\,s + 25\,s^2}$



Bode Plot: Multiplication Rules

Multiplication of Transfer Functions

$$G(s) = G_1(s) \cdot G_2(s) \cdot \dots \cdot G_n(s)$$

Addition of Magnitude and Phase in Bode Plot

- $|G(j\omega)|_{dB} = |G_1(j\omega)|_{dB} + |G_2(j\omega)|_{dB} + \cdots + |G_n(j\omega)|_{dB}$
- $\angle G(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega) + \cdots + \angle G_n(j\omega)$

Gap 4

Bode Plot: Transfer Function Zeros

Inverse of Transfer Functions

$$G(s) = G_1^{-1}(s)$$

Negation of Magnitude and Phase

- $|G(j\omega)|_{dB} = -|G_1(j\omega)|_{dB}$
- $\angle G(j\omega) = -\angle G_1(j\omega)$

Gap 5

Bode Plot: Non-minimum Phase Factors

First-order Lag Example

$$G(s) = \frac{1}{1 - T\,s}$$

Comparison to Minimum-Phase Factor

- $|G(j\omega)|_{dB} = |\frac{1}{1 + j\omega\,T}|_{dB}$; $\angle G(j\omega) = -\angle(\frac{1}{1 + j\omega\,T})$

Gap 6

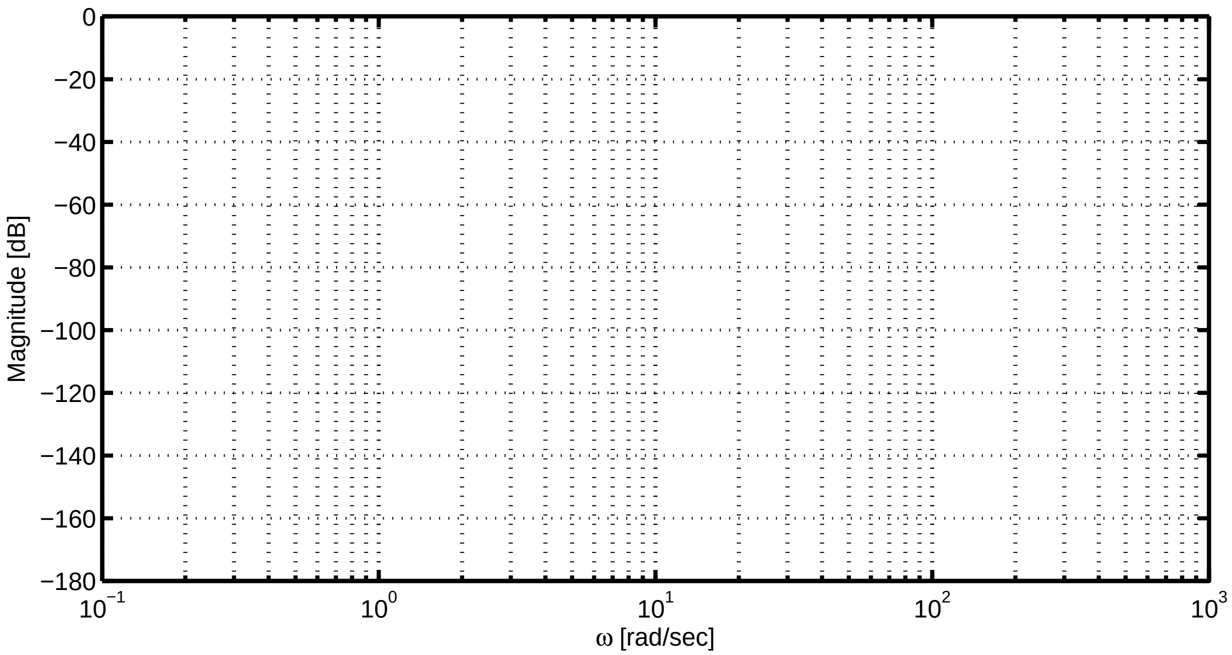
Bode Plot: Example $G(s) = \frac{5}{(s+5)(s^2+0.2\,s+1)}$

Computation

Gap 7

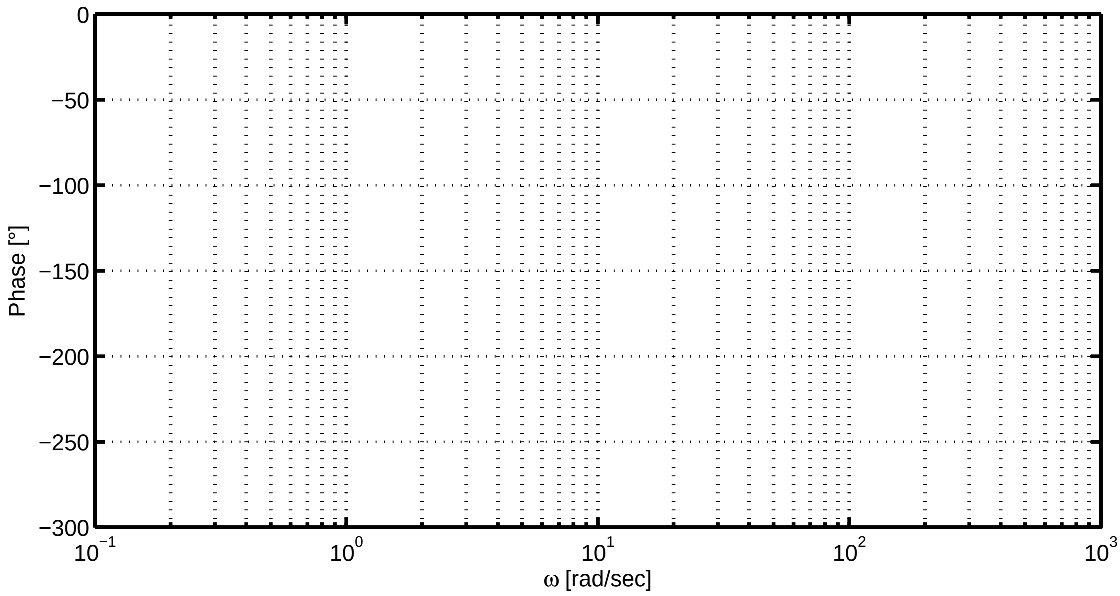
Bode Plot: Example $G(s) = \frac{5}{(s+5)(s^2+0.2\,s+1)}$

Magnitude Plot



Bode Plot: Example $G(s) = \frac{5}{(s+5)(s^2+0.2\,s+1)}$

Phase Plot



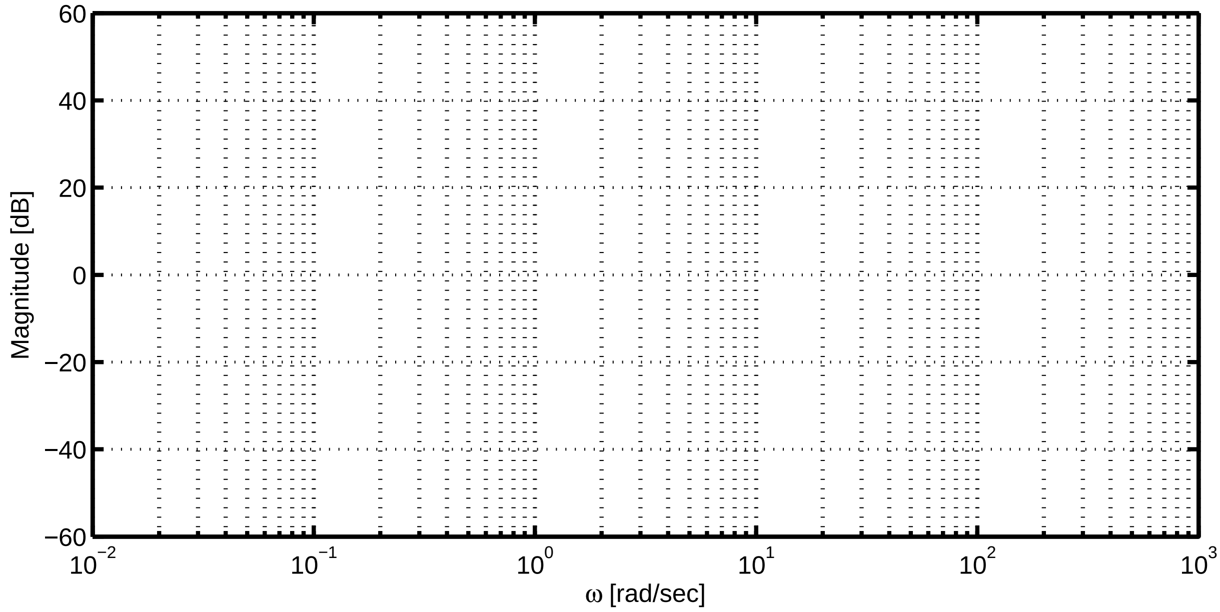
Bode Plot: Example $G(s) = \frac{s+10}{s\,(s+1)}$

Computation

Gap 8

Bode Plot: Example $G(s) = \frac{s+10}{s\,(s+1)}$

Magnitude Plot



Bode Plot: Example $G(s) = \frac{s+10}{s\,(s+1)}$

Phase Plot