Laboratory 13: State Space Control

Problem 28:

We consider a plant with the following state equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_2 & 0 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 0 \\ b_1 \\ 0 \\ b_2 \end{bmatrix} \cdot u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \cdot x(t)$$

Check controllability of the system for the following choices of the parameters a_1, a_2, b_1, b_2.

	a_1	a_2	b_1	b_2
Case 1	-9	25	17	-9
Case 2	0	20	15	0
Case 3	9	25	17	9

Problem 29:

We revisit the active vehicle suspension system from Problem 1 and 4. We recall the state equations

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{K \cdot A_H}{m} \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

We assume that all parameters c, K, A_H, m are positive constants.

a. Show that the system is instable.

We want to stabilize the system by an appropriate state feedback

b. First check that the system is controllable.

c. Now design a feedback vector k such that the poles of the closed-loop system lie at $s_1 = -5$ and $s_2 = -5$.

d. Find the pre-filter value M such that a reference unit step $r(t) = \sigma(t)$ leads to a final output value of $\lim_{t\to\infty} y(t) = 1$

e. Write down the overall state feedback equation. Which states have to be measured?

f. Simulate a reference step response of the feedback loop. Use $m = 1000\,\text{kg}$, $c = 10\,000\,\text{N/cm}$, $A_H = 15\,\text{cm}^2$, $K = 100\,\text{N/cm}^2/\text{V}$.

g. Repeat **c.** to **e.** for the closed-loop poles $s_1 = -4 + 4\,j$ and $s_2 = -4 - 4\,j$.