Laboratory 2: Transfer Function and Step Response

Problem 3:

Simulate the step response of the following transfer functions.

$$G_1(s) = \frac{5}{s+5} \qquad G_2(s) = \frac{s+4}{s^2+4\,s+1} \qquad G_3(s) = \frac{5}{s-5}$$

Problem 4:

Consider the following state space model with the real parameter $k \in \mathbb{R}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & -k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a. Determine the transfer function for the above state space model

b. For which values of k do you expect no oscillations in the step response?
Hint: Determine the damping D depending on k.

c. For which values of k do you expect decreasing oscillations in the step response?

d. For which values of k do you expect increasing oscillations in the step response?

e. Use Simulink to simulate a step-response of the transfer function in **a.** for the values $k = -1$, $k = 1$ and $k = 10$. Compare your result to **b.**, **c.** and **d.**.

Problem 5:

We recall the following state space model of the vehicle suspension system from Problem 2:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{A_H\,K}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

a. Compute the transfer function of the vehicle suspension system.

b. Use the transfer function in **a.** to simulate the response of the vehicle suspension system to an input step of $10\,\sigma(t)$ (parameters are $m = 1000\,\text{kg}$, $c = 10\,000\,\text{N/cm}$, $g = 10\,\text{N/kg}$, $A_H = 15\,\text{cm}^2$, $K = 100\,\text{N/cm}^2/\text{V}$).

c. What do you observe in comparison to the simulation in Problem 2?

Problem 6 [Self Study]

We are given the following transfer functions

$$G_1(s) = \frac{s+4}{s\,(s+1)} \qquad G_2(s) = \frac{s+4}{s+1}$$

a. Compute the impulse response and the step response for $G_1(s)$.

b. Compute the ramp response for $G_2(s)$.

Hint: First compute $Y(s)$ and then use the inverse Laplace transformation to obtain $y(t)$.