# Laboratory 4: Properties of LTI Systems

## Problem 9:

Consider the following transfer functions.

$$G_1(s) = \frac{s^2+1}{s^5+16\,s^4+6\,s^3+80\,s^2+3\,s+16} \qquad G_2(s) = \frac{(s-4)^2}{s^2+4\,s+3}$$

$$G_3(s) = \frac{s+3}{s^2+3\,s-4} \qquad G_4(s) = \frac{s^2\,(s+1)}{s^2+2\,s+2}$$

**a.** Determine the relative degree of each transfer function. State which transfer function is *strictly proper*, *proper*, *improper*.

**b.** Determine which of the transfer functions is BIBO stable. Use the Routh-Hurwitz test if required.

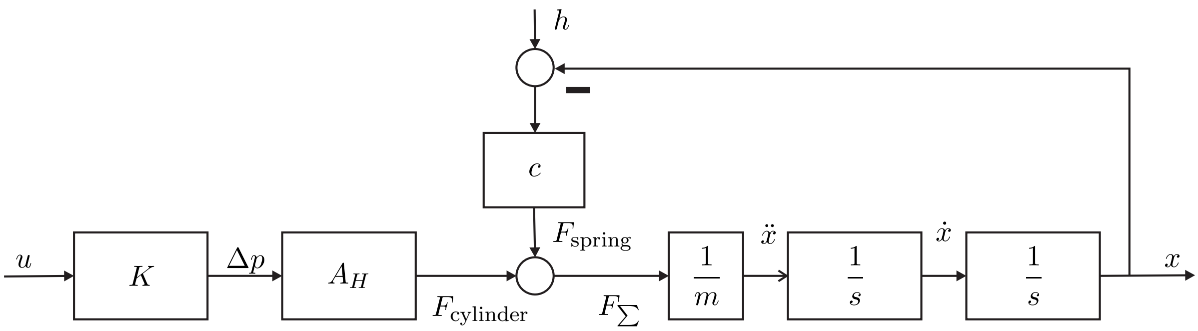
**c.** Sketch the pole/zero diagram for $G_2(s)$, $G_3(s)$ and $G_4(s)$

**d.** Confirm your result in **b.** for $G_1(s)$, $G_2(s)$ and $G_3(s)$ by simulating their step response.

## Problem 10:

**a.** The transfer function of the vehicle suspension system is $G(s) = \frac{K\,A_H}{m\,s^2+c}$. Is the vehicle suspension system BIBO stable?

We recall the block diagram of the vehicle suspension system.




We assume an additional friction force $F_d = -\gamma\,\dot{x}$ with the constant $\gamma$.

**b.** Add the damping force to the block diagram.

**c.** Show that the modified vehicle suspension system is BIBO stable for all $\gamma > 0$.

**d.** Confirm your results by simulating output responses of the original and the modified vehicle suspension system for the input signal $u(t) = \sin(\sqrt{10}\,t)$. Use $\gamma = 2000\,\text{N sec/cm}$.

Hint: You can find the simulink model of the original vehicle suspension system on the course webpage.

Recall the parameters $m = 1000\,\text{kg}$, $c = 10\,000\,\text{N/cm}$, $g = 10\,\text{N/kg}$, $A_H = 15\,\text{cm}^2$, $K = 100\,\text{N/cm}^2/\text{V}$.

**e.** Also simulate the response of the modified vehicle suspension system to an input step of $u(t) = 10\,\sigma(t)$. Do you expect real or complex poles of the transfer function in **c.**?